

Phantom Energy and the Cosmic Horizon: R_h is still not a horizon!

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ABSTRACT

There has been a recent spate of papers on the Cosmic Horizon, an apparently fundamental, although unrecognised, property of the universe. The misunderstanding of this horizon, it is claimed, demonstrates that our determination of the cosmological makeup of the universe is incorrect, although several papers have pointed out key flaws in these arguments. Here, we identify additional flaws in the most recent claims of the properties of the Cosmic Horizon in the presence of phantom energy, simply demonstrating that it does not act as a horizon, and that its limiting of our view of the universe is a trivial statement.

Key words: cosmology: theory

1 INTRODUCTION

The presence of various horizons within our cosmological models have greatly elucidated our understanding of the workings of the universe, with both the particle and event horizons limiting the connexions between past and future cosmological events (Rindler 1956). The universe also possesses a Hubble Sphere, which is not a horizon, and is the distance from an observer that comoving objects are moving, due to the cosmic expansion, at the speed of light (in proper coordinates see Harrison 1991).

There has been the claim that the universe possess another, previously unidentified, horizon, dubbed the Cosmic Horizon (R_h), and the presence of this horizon significantly influences our observations of the cosmos (Melia 2007, 2009; Melia & Abdelqader 2009; Melia & Shevchuk 2012; Bikwa, Melia, & Shevchuk 2012; Melia 2012b), although several authors have demonstrated that a number of the key claims about the Cosmic Horizon are incorrect (van Oirschot, Kwan, & Lewis 2010; Lewis & van Oirschot 2012; Bilicki & Seikel 2012).

In this brief contribution, we discuss the recent claims of R_h in the presence of phantom energy (Melia 2012a), again showing them to be demonstrably incorrect. In Section 2 we briefly review the nature of R_h , while in Section 3 we discuss what R_h means for the path of a photon traveling through an expanding universe. Section 4 demonstrates that R_h still fails to behave as an unrecognised horizon in limiting our view of the universe, and the conclusions are presented in Section 5.

2 THE COSMIC HORIZON

The concept of the Cosmic Horizon, R_h , was introduced by Melia (2007) who, in rewriting the standard Friedmann-Robertson-Walker (FRW) invariant interval in ‘observer-dependent coordinates’ found metric terms that appeared to be singular at a proper distance of $R_h=1/H$, where H is the Hubble constant; it is important to remember that H evolves over cosmic time, and so R_h is a similarly evolving proper distance from an observer. In this initial work, Melia (2007) claimed that the divergence of the metric components showed that R_h represented an infinite redshift surface, and hence a limit to our view of the universe. However, this was shown as being incorrect by van Oirschot, Kwan, & Lewis (2010), who demonstrated that the infinite redshift was due to a unphysical choice for the coordinate velocity of an emitter at a distance of R_h , and correctly accounting for the coordinate transformation between FRW and observer coordinates results in the same redshift; hence we can see photons that have traveled through R_h .

As noted in van Oirschot, Kwan, & Lewis (2010), the un-horizon-like properties come as no surprise as R_h is exactly the same as the Hubble Sphere, a very well understood concept in cosmology (Harrison 1991). While in our current cosmology the Hubble Sphere will eventually become coincident with our event horizon (see Figure 1 of Davis & Lineweaver 2004), it is not, in itself, a horizon.

However, there have been continuing claims about the fundamental nature of R_h , and recently, Bikwa, Melia, & Shevchuk (2012) considered the paths of photons over cosmic history, examining the proper distance traversed since the Big Bang. In considering several standard cosmological models, they concluded that the fundamental property of R_h is that any photon we receive today cannot have traveled

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from a distance greater than R_h today. However, this was also shown to be incorrect by Lewis & van Oirschot (2012) who demonstrated that R_h does not have to continuously grow or asymptote to a particular distance, and that if dark energy is actually of the form of phantom energy (with an equation of state of $\omega < -1$), then R_h can decrease. Hence, light rays arriving at an observer can have travelled from a larger proper distance than R_h today (see Figure 3 of Lewis & van Oirschot 2012).

The question of the influence of phantom energy on R_h was revisited in Melia (2012a), who again redefined the cosmological properties of this Cosmic Horizon, concluding two key features of the photon paths should reassure us of its fundamental importance. These are that;

“The most important feature of these curves is that none of those actually reaching us [today] ever attain a proper distance greater than the maximum extent of our cosmic horizon.”

and

“every null geodesic that possesses a second turning point, diverges to infinity”

In the remainder of this paper, we will demonstrate that the first assertion is trivial, and the second is incorrect.

3 TO A PHOTON, JUST WHAT IS R_h ?

As we have noted previously, R_h simply corresponds to the Hubble Sphere (van Oirschot, Kwan, & Lewis 2010; Lewis & van Oirschot 2012), the distance from an observer at which the universal expansion results in a proper velocity of the speed of light for a comoving object. In this section, we will discuss the what passing through the Hubble Sphere means to a photon, although it should be noted that this has been discussed in detail previously (e.g. Ellis & Rothman 1993).

Starting with the FRW invariant interval for a spatially flat expanding space-time,

$$ds^2 = -dt^2 + a(t)^2 (dr^2 + r^2 d\Omega^2) \quad (1)$$

where r is the comoving radial coordinate, $a(t)$ is the time-dependent scale factor, and $d\Omega$ considers the angular coordinates. With this, the proper distance to an object at a comoving distance of r is $d = a(t) r$. For an observer at rest with regard to the comoving spatial coordinates, the experienced proper time is the same as the cosmic time, t , and so we can talk of the relative velocity of the distant object with regards to the observer as being

$$\dot{d} = \dot{a}r + a\dot{r} \quad (2)$$

where the dots denote derivatives with regards to t , and the \dot{r} represent motion relative to the comoving coordinates.

If we consider a photon moving in the radial direction, so the angular terms in Equation 1 can be neglected, and remembering that a photon path follows $ds = 0$, then it is straight-forward to show

$$\dot{r} = \pm \frac{1}{a} \quad (3)$$

and substituting the negative solution, as we are interested in a photon approaching the observer, into Equation 2, we find

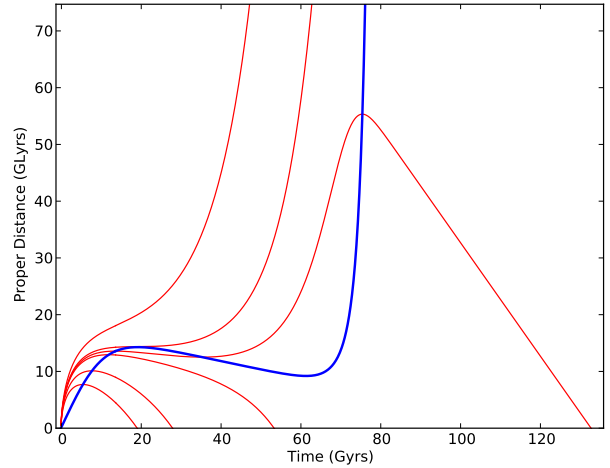


Figure 1. The blue curve presents R_h for the cosmological model outlined in Section 4, while the red lines present a series of light rays emanating from the Big Bang (at the origin) and into the future. Note that the abscissa represent cosmic time, whereas the ordinate is proper distance from an observer (at a proper distance of zero). In this representation, four light rays arrive back at the observer; the key light ray is the one arriving at ~ 130 Gyrs, as this has crossed R_h three times.

$$\dot{d} = \frac{\dot{a}}{a} (ar) - a \left(\frac{1}{a} \right) = Hd - 1 \quad (4)$$

so at proper distances of $d > 0$, the photons are traveling, relative to the observer, at velocities not equal to the speed of light. At the distance of $d = 1/H$, which is R_h (or, in its proper parlance, the Hubble Sphere), then it is simple to see that $\dot{d} = 0$, and the photon is at rest with regards to the observer (in proper coordinates).

What this means is that, in proper coordinates, a photon crossing R_h represents an extremal or inflection point in the photon's path, and, considering that the photon path began at $r = 0$ in the Big Bang, and returns to $r = 0$ at a later time (i.e. it is detected by an observer), there must be a ‘most-distant’ turn-around point in the photon's journey, where it stops heading outward and starts heading inwards, and this must coincide with the photon crossing R_h .

In summary, the claim by Melia (2012a) that R_h represents a bound to the observed photon's path is nothing more than saying “The maximal distance from which we receive a photon is no more than the largest distance at which it turns around in its journey”; this is a trivial statement.

4 EVOLVING HORIZONS

As a demonstration of the meaning of the Cosmic Horizon, we consider a universe in which R_h can be tuned to be increasing or decreasing by modulating the equation of state of the dark energy component. We begin by adopting the cosmological parameters of $\Omega_M = 0.27$, $\Omega_\omega = 0.73$ and $H_0 = 72 \text{ km/s/Mpc}$. Unlike standard cosmological models, we allow the equation of state of the dark energy component, ω , to change as a function of time, adopting an evolutionary form given by

$$\omega(t) = -1.1 + \frac{1.43}{1 + \exp\left(\frac{6-t_h}{0.3}\right)} \quad (5)$$

where $t_h = t/(13.58 \text{ GYrs})$ is the scaled cosmic time; it should be noted that this evolution is not physically motivated and simply provides a model in which we can examine the corresponding evolution of R_h . With this form, the transition in the form of the dark energy component takes place at $t \sim 6t_h$, and the duration of the change over is $\sim 0.3t_h$, although the values were chosen for purely illustrative purposes. Essentially, in the early universe, the dark energy component has an equation of state of $\omega \sim -1.1$, corresponding to a phantom energy, while around $t \sim 5t_h \sim 65 \text{ GYrs}$ the equation of state begins to transition to $\omega \sim \frac{1}{3}$, representing the equation of state of a relativistic mass-less fluid, such as photons (see Linder 1988). Note that given this evolution, expansion means that the universe will become matter dominated in the future, given that the energy density in the now photon-like dark energy component diminishes faster with cosmological expansion.

In Figure 1, we present the key properties of this universe in terms of the cosmic time (abscissa) and proper distance (ordinate). The blue curve denotes the evolution of R_h , whereas the red curves represent light rays which emanate from the Big Bang (at the origin) and travel into the future. The left-hand half of the plot can be compared directly with Figure 3 of Lewis & van Oirschot (2012), with R_h increasing when the universe is matter dominated, and then decreasing as the phantom energy comes to dominate. However, as the equation of state of the dark energy component evolves towards $\omega \sim \frac{1}{3}$, then R_h rapidly increases.

An examination of the light rays in Figure 1 show that, in this representation of the universe, all photons head outwards after the Big Bang. Following this initial motion, three of the photon paths then encounter the evolving R_h only once, and then arrive back at the observer; this can be simply understood as, in proper coordinates, R_h marks the turning point in a photon's path. One photon path in Figure 1 does not encounter R_h , so that there is no turning point in its path and it does not return to the observer.

There are two additional light rays which cross R_h more than once. Again, these rays are initially heading away from the origin after the Big Bang, and both of them cross R_h and begin to head back towards the observer. However, due to the presence of the phantom energy component, both photon paths encounter a decreasing R_h and the motion is reversed and the paths again move away from the observer; note that while one of these paths appear have a point of inflection, just touching R_h , it actually does possess two crossings. As the equation of state of the dark energy component is changing and influencing the cosmic expansion, while one photon path escapes to larger distance, the other is again reversed, at a point where the photon passes through R_h and it now continues its journey towards the observer, arriving at a cosmic time of $t \sim 130 \text{ GYrs}$. This is in stark contrast to the claim made by Melia (2012a).

It is important to note that the results presented in this paper do not depend upon the specific form of the evolving dark energy described by Equation 5. If we consider a universe with single energy component described by an equation of state ω , then R_h evolves as

$$\dot{R}_h = \frac{3}{2} (1 + \omega) \quad (6)$$

and hence for phantom energies, with $\omega < -1$, R_h would decrease, for a cosmological constant ($\omega = -1$) it is a constant, and all other fluids, with $\omega > -1$, R_h increases. If Equation 5 is modified so that the ultimate equation of state of the dark energy component is $\omega = 0$, corresponding to matter, R_h will increase in the future, similar to the evolution shown in Figure 1. Again outward moving light rays encountering this increasing R_h will change direction and will head back to the observer.

An examination of the discussion presented here should convince the reader that by modifying the equations of state of the energy components of the universe, we could ensure that R_h oscillates through a arbitrarily complex path, and similarly that photon journeys can be made to arbitrarily change their direction of motion toward or away from an observer. Each change of direction is accompanied with the photon crossing in and out of R_h , which is extremely un-horizon-like behaviour (but precisely what you would expect as R_h being a turning point in a photon's path).

5 CONCLUSIONS

We have examined the most recent claims of the fundamental nature of the Cosmic Horizon, R_h , made by Melia (2012a), especially its behaviour in the presence of phantom energy. We have demonstrated that, by modifying the equation of state of the energy components in the universe, then R_h can be made to grown and shrink arbitrarily. Furthermore, as R_h represents the location of where a photon changes direction, or goes through an inflection point, (in proper coordinates) relative to an observer, an oscillating photon path can cross R_h a multitude of times before arriving at an observer. The 'fundamental' property of R_h in terms of the farthest proper distance from which an observer receives a photon is essentially a trivial statement.

As we have stressed in other contributions (e.g. Lewis & van Oirschot 2012), the importance and evolution of true cosmic horizons is well understood, as is the mean of the Hubble Sphere (Harrison 1991). Other than the trivial, the Cosmic Horizon of Melia (2012a) does not present a fundamental limit to our view of the universe.

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